3.7b Implicit Differentation

Show that dy/dx is defined at every point on the graph of $2y = x^2 + \sin(y)$

$2y' = 2x + y'\cos(y)$

$2y' - y'\cos(y) = 2x$

$y'(2 - \cos(y)) = 2x$

$y' = \frac{2x}{2 - \cos(y)}$

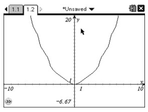
undefined if

$2 - \cos(y) = 0$

$2 = \cos(y)$

can't happen

max of $\cos(y)$ is 1



Graph the curve using parametric equations

solve for x: $x^2 = 2y - \sin(y)$

$x = \pm\sqrt{2y - \sin(y)}$

$x_1 = \sqrt{2t - \sin(t)}$

$y_1 = t$

$x_2 = -\sqrt{2t - \sin(t)}$

$y_2 = t$

Sep 29-7:40 AM

$x^2 - 2xy + y^2 = 4$

1. Find dy/dx

$2x - (2xy' + 2y) + 2yy' = 0$

$2x - 2xy' - 2y + 2yy' = 0$

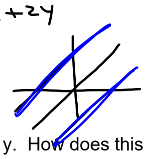
$-2xy' + 2yy' = -2x + 2y$

$y'(-2x + 2y) = -2x + 2y$

$y' = \frac{-2x+2y}{-2x+2y} = 1$

$y' = 1$

2. Use dy/dx to sketch a possible graph of the implicit curve.



3. Factor the left side and solve for y. How does this compare with your graph?

$(x-y)(x-y) = 4$

$(x-y)^2 = 4$

$x - y = \pm 2$

$y = x + 2$ or $y = x - 2$

Sep 29-8:00 AM

Prove the power rule for rational exponents

$y = x^{2/3} = \sqrt[3]{x^2}$

$y' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$

$y = \sqrt[5]{(x+1)^3} = (x+1)^{\frac{3}{5}}$

$y' = \frac{3}{5}(x+1)^{-2/5} = \frac{3}{5\sqrt[5]{(x+1)^2}}$

$\frac{d}{dx}\sqrt[3]{5x^2} = \frac{d}{dx}(5^{1/3}x^{2/3}) = 5^{1/3} \cdot \frac{2}{3}x^{-\frac{1}{3}}$

Sep 29-8:12 AM

Find the slope of the Folium of Descartes at the points (4,2) and (2,4).

$x^3 + y^3 - 9xy = 0$

$3x^2 + 3y^2y' - (9xy' + 9y) = 0$

$3x^2 + 3y^2y' - 9xy' - 9y = 0$

$3y^2y' - 9xy' = -3x^2 + 9y$

$y'(3y^2 - 9x) = -3x^2 + 9y$

$y' = \frac{-3x^2 + 9y}{3y^2 - 9x}$

$y' = \frac{-x^2 + 3y}{y^2 - 3x}$

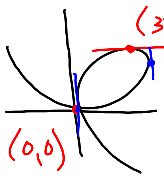
$y'(4,2) = \frac{-4^2 + 3 \cdot 2}{2^2 - 3 \cdot 4} = \frac{-10}{-8} = \frac{5}{4}$

$y'(2,4) = \frac{4}{5}$

Find the points where the folium has: (a) a horizontal tangent; (b) a vertical tangent

a) top = 0 $\quad -3x^2 + 9y = 0$

$(3\sqrt[3]{2}, 3\sqrt[3]{4})$



(0,0)

solve $\begin{cases} x^3 + y^3 - 9xy = 0 \\ -3x^2 + 9y = 0 \end{cases}$

b) bottom = 0

Sep 29-8:15 AM

Sep 26-10:24 AM